Simulation of Learning Processes with the SIR Model: Interaction and Motivation

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Abstract: The classical SIR model has been modified by analogizing the spread of diseases to the transmission of information, and the Student Learning Behavior (SLB) model has been developed. This study emphasizes the significant impact of students' learning behavior and motivation. It also highlights the critical role of motivation by examining the effect of students' learning behaviors on academic achievement. By adapting the SIR epidemiological model to learning processes, the study aims to investigate the transmission of knowledge among students, interaction dynamics, and motivational factors. Using the characteristics of the SIR model, it demonstrates how the spread of learning can be mathematically represented. The influence of motivation on students' learning processes is discussed. The study and its findings provide an important and distinct framework for researchers who examine the dynamic nature of learning processes from both conceptual and quantitative perspectives.

Keywords: Student Learning Behavior (SLB); Motivation; SIR model; Cellular automaton

I. Introductıon

Mathematical models are powerful tools that today have wide application areas in many fields such as science, engineering, and social sciences. These models represent a system using mathematical concepts and enable the analysis of system behavior under various conditions (Setyawan et al., 2001). They are widely used to predict the behavior of complex systems and to examine their responses to different conditions. Particularly in the field of epidemiology, the SIR (Susceptible–Infected–Recovered) model, developed by Kermack and McKendrick and forming the foundation of modern epidemiological modeling, is defined as a compartmental model for studying disease spread. In these models, the population is divided into subgroups with specific characteristics, and changes over time are described using differential equations (Setyawan et al., 2001).

The integration of Cellular Automata (CA) and SIR models has provided an important analytical framework for simulating dynamic processes involving interaction, diffusion, and behavioral change. Initially, CA-based epidemic models were developed to simulate the spatial and temporal heterogeneity of infectious diseases with greater accuracy compared to traditional differential equation models. Studies in this area have shown that CA can model the spread of infection at the population level with high accuracy by considering parameters such as population density, mobility, and interaction distance (Mikler, Venkatachalam, & Abbas, 2005; Fuks & Lawniczak, 2002). Subsequent extensions have deepened these approaches through stochastic lattice gas and SIRS+V models incorporating probabilistic behavior and vaccination dynamics (Ariel et al., 2016; Shabunin, 2023). Schneckenreither et al. (2008) compared SIR-type epidemic dynamics using cellular automata and differential equations and found that CA models better represent local interactions.

In recent years, the application area of the SIR model has expanded beyond biological epidemics to include social sciences and educational research. Bettencourt et al. (2006) demonstrated that the spread of information or behavior can also be represented using the SIR model. In this context, researchers have begun modeling learning processes using SIR dynamics by analogizing information diffusion to an infection process. This approach indicates that student interactions, motivation, and engagement are factors influencing the “contagion” of knowledge. Kumar (2025) revealed this relationship by developing the Student Learning Behavior (SLB) model, which defines the learning process influenced by peer interaction and self-motivation as an epidemic. The SLB model simulated knowledge acquisition and retention by likening student learning behaviors to epidemic modeling and demonstrated that social interaction and self-motivation play decisive roles in learning success. This model offers a more standardized classical framework for student learning behavior based on biological epidemic and SIR models. Similarly, a CA-based model developed by Liang, Liu, Zhang, and Yang (2010) emphasized the effects of motivation and learning environment on students’ behavioral dynamics.

In parallel, modified CA–SIR models have been widely used to simulate emotional and social interaction processes. Studies by Fu et al. (2014) and Xue et al. (2019) showed that emotional contagion within crowds or groups can be modeled with infection-like dynamics, and that factors such as personality traits, environmental stress, and social density influence these processes. These findings support conceptualizing learning as a social diffusion process, suggesting that knowledge, attitudes, and motivation spread through interpersonal interaction. Recent research has further advanced these analogies by developing hybrid CA–SIR models. For example, the FDE-LLM framework combines epidemic modeling with artificial intelligence–based systems to more accurately simulate the spread of opinions and information in social networks (Yao et al., 2025). Similarly, CA-based network models developed by Kyriakou et al. (2022) examined the role of structural properties, mobility, and interaction in diffusion processes, highlighting the importance of network topology in information or behavior spread.

Epstein (2006) integrated the SIR model into agent-based models to examine the effects of individual behaviors on epidemic dynamics. Hajary and Ahmadi (2014) demonstrated, using cellular learning automata, that collaborative interactions among learners increase the speed and depth of knowledge acquisition. Oommen and Hashem (2010) modeled classroom learning as an interactive automata system and revealed that weaker students could improve their learning performance by up to 73% through collaboration with peers. The convergence of these research directions shows that learning, like disease or emotional spread, is a complex adaptive process. Individual factors (e.g., motivation), interpersonal transmission, and the structural properties of interaction networks shape the collective dynamics of learning.

The complex nature of learning processes requires consideration not only of cognitive factors but also of motivational and emotional factors. Pekrun et al. (2002) examined the effects of academic emotions on self-regulation and achievement using qualitative and quantitative methods. Boekaerts

(1997) proposed a self-regulated learning model integrating metacognition and motivation. Schunk (1991) explained the decisive role of self-efficacy beliefs in academic motivation. Wolters (2003) addressed students' motivation regulation strategies and their effects on learning. Artino (2009) investigated the role of motivation and self-regulation in online learning. Mega, Ronconi, and De Beni (2014) presented a holistic model evaluating the combined effects of emotions, motivation, and self-regulation on achievement.

The central role of motivation in learning is also evident in the teacher–student motivation management model developed by Zaikin et al. (2016), which showed that interactive motivational processes strengthen collaboration and participation. Damanik et al. (2019) stated that learning motivation interacts with collaborative learning models to have a significant impact on academic achievement. Abdelrahman and Hidayat (2020) investigated the effects of academic motivation and metacognitive awareness on the academic achievement of Ajman University students. Al-Osaimi and Fawaz (2022) proposed considering learning as a fundamental principle for students' academic success. Chen and Hastedt (n.d.) examined the impact of student motivation on international assessment results and academic achievement.

Student learning depends on many factors related to learners' conditions, such as family environment, peer groups, school, and social influences. Many students experience family environments that do not support their needs; while they wish to be productive, they are affected by various difficulties. Such disruptions are significant factors that slow learning pace and negatively affect motivation and learning ease (Abdelrahman & Hidayat, 2020). Kumar (2025) noted that teaching methodology also plays an important role in reducing students' interest in lessons. Teachers identify various problems in classroom environments, such as being assigned additional duties considered more important than teaching and the inability of a single teacher to attend equally to all students in crowded classrooms. While students sitting in the front rows receive more attention, those in the back receive less, and learning slows when teachers fail to provide adequate attention.

As a result, many students face learning difficulties. Students who think more slowly than their peers and have lower motivation are often defined as "slow learners" (Kishore et al., 2022). These students have below-average IQ levels and struggle to meet teacher and parental expectations because they cannot progress at the same pace as their peers. Malik (2009), Shaw (2010), and Chauhan (2011) examined assessment procedures for mathematics students with very slow learning processes. Dasarahdi et al. (2016) showed through a literature review that many analysts have used epidemic models to demonstrate that certain curriculum activities directly or indirectly affect students' academic achievement and overall development. Mutiawati et al. (2022) investigated the improvement of learning behaviors of slow learners. In education, motivation-based learning processes have been examined within inclusive education and categorized into four groups: uninterested individuals, slow learners who do not disturb others, learners who disturb others, and developing learners (Kumar, 2025). Demir (2024) demonstrated that student learning behavior can be analyzed using the SEIR model, typically used to study infectious diseases, and showed that this approach enables a deeper understanding of students' learning behaviors.

All these studies reveal that modeling learning as a cellular automata–based SIR system effectively captures the complex relationship among social interaction, motivation, and knowledge diffusion. Cellular automata models integrated with extended SIR frameworks offer a powerful method for simulating learning processes and analyzing the effects of motivation and interaction on learning diffusion. In this study, the SIR model is considered not only as an epidemiological tool but also as a method for understanding and analyzing the dynamic universe of learning processes. The integration of an extended SIR model with cellular automata simulation enables a more comprehensive examination of the roles of student interaction and motivation in learning processes.

II. Materıals and Methods

A. Adaptation of the SIR Model to Learning Processes

Learning processes involve not only the acquisition of knowledge but also stages such as receiving, processing, applying, and transferring information. Therefore, the classical SIR model may be insufficient to fully explain learning-related parameters. To address this limitation, the model is extended and a four-/multi-component structure is proposed.

Table 1 systematically defines the key parameters and state variables used in adapting the extended SIR model to learning processes, thereby clarifying the conceptual framework of the model. In the model, β represents the extent to which motivation spreads among students, while λ denotes the initial rate of active engagement in the learning process. The parameter α, which expresses the influence of peers and expert learners in knowledge transfer, highlights the decisive role of social interaction in learning dynamics. The parameter δ represents the gradual loss of motivation over time, describing a fundamental mechanism that limits the sustainability of learning behavior. The parameter τ explains how the guiding influence of teachers and experienced learners in transferring knowledge and motivation is incorporated into the model.

In this model, the state variables $U(t)$, $M(t)$, $L(t)$, $T(t)$, and $D(t)$ represent different student groups as follows:

$U(t)$: Uninformed;
$M(t)$: Motivated;
$L(t)$: Learning;
$T(t)$: Mastered;
$D(t)$: Low motivation.

The total total number of students is given by .

These state variables represent students' different positions along the learning journey and enable tracking the system's behavior over time. This terminological structure provides a solid foundation for both the mathematical formulation of the model and the dynamic analysis of learning processes.

Table 1 Terminology

Parameter / Variable	Description
β	Motivation spread
λ	Learning initiation rate
α	Peer/expert influence
δ	Motivation loss
τ	Teacher/experienced learner influence
$U(t)$	Uninformed
$M(t)$	Motivated
$L(t)$	Learning
$T(t)$	Mastered
$D(t)$	Low motivation

The primary objective of this study is to provide a framework that strengthens logical and critical thinking skills by examining the role of motivation in the learning process through mathematical and computational methods. Using the SIR model, learning behavior is characterized, and a qualitative outcome of motivation and interaction is developed. Social interaction and logical (critical) thinking are fundamental goals of the learning process. Group work and peer interaction play an effective role in increasing the self-confidence of slow-learning students.

Figure 1 presents the extended SIR approach adapted to learning processes by illustrating the flow relationships and conceptually integrating how students transition between different learning states. In the model, $U(t)$ represents uninformed individuals or those not yet engaged in the learning process; this group can gain motivation through the parameter β and transition to the $M(t)$ class. $M(t)$ denotes motivated students, who enter the learning initiation phase and move to the $L(t)$ (learning) group under the influence of λ. Students at the $L(t)$ level can become $T(t)$ (mastered) individuals through peer or expert influence represented by α. At the same time, the parameter 2δ, which expresses motivation loss, explains the transition of learners to the $D(t)$ (low motivation) class. Individuals in the $T(t)$ group may also transition to the $D(t)$ state due to motivation loss; conversely, they can support the $L(t)$ level through feedback interactions with learners.

This flow structure emphasizes that learning and motivation constitute a nonlinear and multidirectional process, clearly reflecting the dynamic nature of the model.

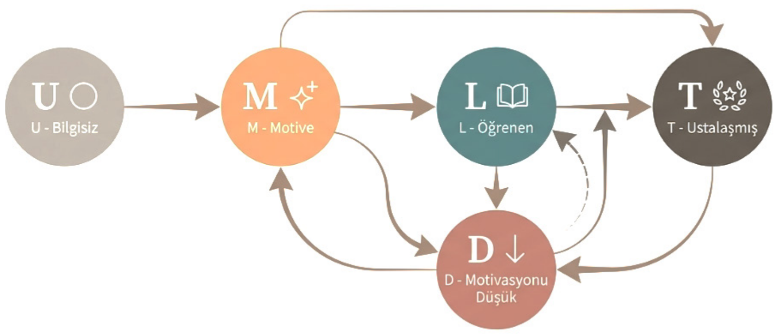

Figure 1 Model scheme

B. Mathematical Formulation of the Model

In this section, a system of differential equations based on an SEIR-type model is presented for students influenced by motivation. The differential equation system used in this study mathematically demonstrates how learning processes evolve on the basis of motivation, interaction, and mastery dynamics.

The expression given in Equation (1) explains the decrease over time of the uninformed student group $U(t)$ as a result of interactions between the parameters β, representing the spread of motivation, and τ, representing teacher/learner influence, with their respective proportions within the total population.

Equation (2) defines the dynamics of motivated students $M(t)$, illustrating multidirectional flows that include both transitions from $U(t)$ through motivation gain and regression toward the $D(t)$ group due to motivation loss δ.

The evolution of the learning group $L(t)$ is addressed in Equation (3) and is modeled through the balance among λ, which characterizes the transition from motivated individuals to learning, α, reflecting peer or expert influence, and 2δ, representing motivation loss.

Equation (4), which explains the behavior of mastered individuals $T(t)$, incorporates both the peer/expert influence α that enables learners to reach mastery and the dissolution dynamics associated with motivation loss.

Finally, the time-dependent change of the low-motivation group $D(t)$ is modeled by Equation (5). This equation defines inflows resulting from motivation loss and transitions back to other classes through re-motivation, thereby revealing the feedback nature of the system.

Taken together, this five-compartment structure demonstrates that learning behavior operates as a nonlinear, multi-interactive, and parameter-sensitive process.

$$\frac{\mathrm{d}t}{\mathrm{d}U} = -\,U\left(\beta\frac{N}{M}+\tau\frac{N}{L}\right) \quad (1)$$

$$\frac{\mathrm{d}t}{\mathrm{d}M} = U\left(\beta\frac{N}{M}+\tau\frac{N}{L}\right) - M\lambda - M\delta\frac{N}{D} + D\beta\frac{N+L}{M} \quad (2)$$

$$\frac{\mathrm{d}t}{\mathrm{d}L} = M\lambda - L\alpha\frac{N}{T} + T - L(2\delta)\frac{N}{D} + 0.5\delta T\frac{N}{D} \quad (3)$$

$$\frac{\mathrm{d}t}{\mathrm{d}T} = L\alpha\frac{N+L}{L} - 0.5\delta T\frac{N}{D} \quad (4)$$

$$\frac{\mathrm{d}t}{\mathrm{d}D} = M\delta\frac{N}{D} + L(2\delta)\frac{N}{D} - D\beta\frac{N+L}{M} \quad (5)$$

C. Model Analysis

The aim of this section of the model analysis is to show that the total population ($N(t)$) is bounded from above over time and that the system is positively invariant, meaning that the solutions do not leave the physically meaningful region of the state space (6). First, the total number of students $N(t)$ is defined as the sum of all subgroups: equation (7).

Taking the time derivative of this total, the derivatives representing the transition rates of each subgroup are summed to obtain ($\frac{\mathrm{d}N}{\mathrm{d}T}$) (8).

An examination of the model equations shows that all transitions between these subgroups balance each other; that is, the internal flows cancel out, resulting in a net internal flow of zero. Consequently, the total population remains constant over time and is bounded, which confirms that the system is positively invariant. This ensures that all state variables remain nonnegative and within a physically meaningful domain throughout the evolution of the system.

$$\lim_{t\to\infty} N(t) \le \frac{T}{\mu} \quad (6)$$

From the model, the total number of students is: N

$$N = U + M + L + T + D$$

$$N(t) = U(t) + M(t) + L(t) + T(t) + D(t) \quad (7)$$

Taking the derivative with respect to (t):

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\mathrm{d}U}{\mathrm{d}t} + \frac{\mathrm{d}M}{\mathrm{d}t} + \frac{\mathrm{d}L}{\mathrm{d}t} + \frac{\mathrm{d}T}{\mathrm{d}t} + \frac{\mathrm{d}D}{\mathrm{d}t} \quad (8)$$

Taking into account that all transitions in the model equations cancel each other out reciprocally (i.e., the internal flows sum to zero), we find that the system has an external inflow only through the term T, and an outflow from the system occurs in the form μN. Thus, using Figure (1), we obtain the following (9):

$$\frac{\mathrm{d}N}{\mathrm{d}t} + \mu N = T \quad (9)$$

This is a linear first-order partial differential equation.

By solving this equation:

$$\frac{\mathrm{d}N}{\mathrm{d}t} + \mu N = T$$

Standard form solution:

$$N(t) = \frac{T}{\mu} + c_1 \mathrm{e}^{-\mu t} \quad (10)$$

Initial value: $N(0) = N_0$

$$N(0) = \frac{T}{\mu} + c_1 \rightarrow c_1 = N_0 - \frac{T}{N} \quad (11)$$

$$N(t) = \frac{T}{\mu} + \left(N_0 - \frac{T}{\mu}\right)\mathrm{e}^{-\mu t} \quad (12)$$

Rewriting, we obtain:

$$N(t) = \frac{T}{\mu}(1 - \mathrm{e}^{-\mu t}) + N_0 \mathrm{e}^{-\mu t} \quad (13)$$

Dolayısıyla:

$$\lim_{t\to\infty} N(t) \le \frac{T}{\mu}$$

This is obtained. This result shows that our system is upper bounded, that is, bounded.

This finding provides an important mathematical guarantee regarding the fundamental structure of the model. The limiting behavior of Equation (13) indicates that the total population approaches a specific upper bound over time namely $\frac{T}{\mu}$ and does not exceed this value. In other words, regardless of how complex the internal flows within the system are, the number of students does not grow uncontrollably; instead, it is constrained by the balance between external inflow (T) and outflow (μN).

This result not only confirms that the model remains within a physically meaningful region but also demonstrates that learning, motivation, and interaction dynamics guide the entire population toward a well-defined equilibrium point. Consequently, the system is mathematically well-posed, and its long-term behavior is predictably bounded, providing a stable foundation for cellular automata simulations.

The admissible solution set of our model is given as follows:

$$\Phi = \begin{Bmatrix} (U, M, L, T, D)\epsilon_{\mathbb{R}\geq 0}^{5}: U \geq 0, M \geq 0, L \geq 0, T \geq 0, D \geq 0, \\ N = U + M + L + T + D \leq \frac{T}{\mu} \end{Bmatrix} \quad (14)$$

When our system is examined from the perspective of Figure (1), this region is positively invariant. Therefore, the model is well defined both mathematically and epidemiologically.

1. Basic Reproduction Number (R_E)

To compute the basic reproduction number, the Next Generation Method (NGM)—the most commonly used approach—is employed. This method is defined based on the infectious compartments of the model. In our model, the "spread of knowledge/motivation" is treated as an infectious process and occurs through the *M* (Motivated) and *L* (Learning) classes.

Therefore, the infectious compartments are:

$$x = \begin{bmatrix} M \\ L \end{bmatrix} \quad (15)$$

It has the form:

From the two relevant equations of the model, the terms responsible for transmission are given as follows:

$$\frac{dM}{dt} = U\left(\beta\frac{N}{M} + \tau\frac{N}{L}\right) - M\left(\lambda + \delta\frac{D}{N}\right) \quad (16)$$

$$\frac{dL}{dt} = M\lambda - L\left(\alpha\frac{L+T}{N} + 2\delta\frac{D}{N}\right) \quad (17)$$

Each equation of the system is decomposed into the following general form:

$$\frac{dx}{dt} = F(x) - V(x) \quad (18)$$

The matrix (*F*), representing the transmission terms, is the linearized transmission matrix obtained by assuming ($U \approx N$, $D \approx 0$) at the initial stage:

$$\boldsymbol{F} = \begin{vmatrix} \beta & \tau \\ \lambda & 0 \end{vmatrix} \quad (19)$$

In the model, motivation loss and learning loss are grouped as follows:

$$V = \begin{vmatrix} \lambda + \delta & 0 \\ 0 & \alpha + 2\delta \end{vmatrix} \quad (20)$$

This matrix is invertible, and its inverse is:

$$V^{-1} = \begin{vmatrix} \frac{1}{\lambda+\delta} & 0 \\ 0 & \frac{1}{\alpha+2\delta} \end{vmatrix} \quad (21)$$

It is obtained in the following form. The Next Generation Matrix (NGM) is defined as:

$$K = FV^{-1} \quad (22)$$

$$K = \begin{vmatrix} \beta & \tau \\ \lambda & 0 \end{vmatrix} \begin{vmatrix} \frac{1}{\lambda+\delta} & 0 \\ 0 & \frac{1}{\alpha+2\delta} \end{vmatrix} \quad (23)$$

The following equality is obtained as a result of the multiplication:

$$K = \begin{vmatrix} \frac{\beta}{\lambda+\delta} & \frac{\tau}{\alpha+2\delta} \\ \frac{\lambda}{\lambda+\delta} & 0 \end{vmatrix} \quad (24)$$

The largest eigenvalue of the matrix (*K*) that we obtained defines the value of the basic reproduction number (R_E). Now, the characteristic matrix of the matrix is given by: FV^{-1}

$$\det(k - \lambda I) = 0 \quad (25)$$

$$\det \begin{vmatrix} \frac{\beta}{\lambda+\delta} - \lambda & \frac{\tau}{\alpha+2\delta} \\ \frac{\lambda}{\lambda+\delta} & -\lambda \end{vmatrix} = 0 \quad (26)$$

$$\begin{vmatrix} \frac{\beta}{\lambda+\delta} - \lambda & \frac{\tau}{\alpha+2\delta} \\ \frac{\lambda}{\lambda+\delta} & -\lambda \end{vmatrix} = \left(\frac{\beta}{\lambda+\delta} - \lambda\right)(-\lambda) - \left(\frac{\tau}{\alpha+2\delta}\right)\left(\frac{\lambda}{\lambda+\delta}\right) = 0 \quad (27)$$

A more simplified form:

$$-\lambda\left(\frac{\beta}{\lambda+\delta} - \lambda\right) - \frac{\tau\lambda}{(\lambda+\delta)(\alpha+2\delta)} = 0 \quad (28)$$

Rearranging, we obtain:

$$\lambda^2 = \lambda\left(\frac{\beta}{\lambda+\delta} + \frac{\tau}{(\lambda+\delta)(\alpha+2\delta)}\right) \quad (29)$$

$$R_E = \frac{\beta}{\lambda+\delta} + \frac{\tau}{(\lambda+\delta)(\alpha+2\delta)} \quad (30)$$

2. Stability Analysis of Equilibrium Points

The purpose of the *Stability Analysis of Equilibrium Points* section is to determine the long-term behavior of the model's dynamic structure and to mathematically identify whether these states are stable. In this context, based on the flow structure shown in Figure (1), two fundamental equilibrium points of the system are considered:

(a) The normal behavior / low-learning equilibrium, where learning and motivation diffusion do not occur.

(b) The endemic equilibrium, where learning and motivation persist within the population.

To obtain these equilibrium points, the system of differential equations Equation (31) for uninformed individuals, Equation (32) for motivated individuals, Equation (33) for learning individuals, Equation (34) for mastered individuals, and Equation (35) for low-motivation individuals is solved under the condition that the time derivatives are set equal to zero. In this way, the equilibrium values of each subgroup are determined, and the parameter regimes under which the system reaches a stable equilibrium are analytically examined.

To derive the equilibrium points of the model shown in Figure (1), the following system is used:

$$\frac{dU}{dt} = T - \beta UM - \tau UL - \mu U \quad (31)$$

$$\frac{dM}{dt} = \beta UM + \tau UL - \left(\frac{dM}{dt}\lambda + \delta\right)M \quad (32)$$

$$\frac{dL}{dt} = \lambda M - (\alpha + 2\delta)L \quad (33)$$

$$\frac{dT}{dt} = \alpha L - \mu T \quad (34)$$

$$\frac{dD}{dt} = \delta M + 2\delta L - \mu D \quad (35)$$

The Purified Equilibrium Point of Normal Learning Behavior represents a scenario in which there is no diffusion of learning or motivation within the system and, consequently, the level of interaction is minimal. In this case, since the motivated, learning, mastered, and low-motivation groups are assumed to be zero ($M = 0$, $L = 0$, $T = 0$, $D = 0$), the dynamic structure is reduced solely to the behavior of uninformed individuals.

Under this assumption, the model simplifies in accordance with the flow relationships shown in Figure (1), and the differential equation describing the temporal evolution of uninformed individuals is obtained as Equation (36). Imposing the equilibrium condition by setting this derivative equal to zero yields Equation (37), and the solution gives the equilibrium value of the uninformed group as $U^{*}=T/\mu$ Thus, the purified equilibrium point of the system is defined as $E_0 = (T/\mu,0,0,0,0)$ biçiminde tanımlanır (39) which consists solely of uninformed individuals, with all other groups equal to zero. This point represents the fundamental equilibrium state describing the long-term behavior of the model in the absence of learning and motivation diffusion.

At this equilibrium point, there is no spread of learning or motivation, and all individuals remain in a low-interaction state; that is, ($M = 0$, $L = 0$, $T = 0$, $D = 0$) Under these conditions, the system derived from Figure (1) becomes:

$$\frac{dU}{dt} = T - \mu U \quad (36)$$

Equilibrium condition:

$$T - \mu U = 0 \quad (37)$$

From this, we obtain:

$$U^{*} = \frac{T}{\mu} \quad (38)$$

Therefore, the purified equilibrium point is:

$$E_0 = (U^{*}, M^{*}, L^{*}, T^{*}, D^{*}) = (\frac{T}{\mu}, 0, 0, 0, 0) \quad (39)$$

The Normal Endemic Equilibrium Point and the stability of this equilibrium reveal the long-term behavior of the model under conditions where learning and motivation persist in the population. In this analysis, by applying the equilibrium condition in which all time derivatives are set equal to zero, the equilibrium expressions for the learning, mastered, and low-motivation groups are first derived; the corresponding results are given in Equations (40)–(46), respectively.

Next, the expression defining the equilibrium value of uninformed individuals is derived in Equations (47) and (48). This value is then related to the equilibrium state of the learning group and simplified in Equations (49) and (50). The necessary conditions for , which constitutes the critical point of the model, are defined in Equations (51)–(53), and the main equality determining the existence of the endemic equilibrium is obtained in Equations (54) and (55).

This result shows that the system can reach an endemic learning state only when the threshold value given by Equation (56) expressed through the basic reproduction number is greater than zero. Finally, all equilibrium values are explicitly expressed using Equations (57) and (58), thereby clearly defining the conditions for the model's transition to the endemic state from both conceptual and mathematical perspectives.

At this equilibrium point, learning persists in the population, with M^*, L^*, T^*, $D^*>0$ indicating that learning and retention continuously increase at this stage. For the endemic equilibrium point, each derivative is set equal to zero.

Let us derive the expressions for L^* ve T^*, D^*:

$$\frac{dL}{dt}=0 \rightarrow \lambda M^* = (\alpha+2\delta)\, L^* \tag{40}$$

$$L^* = \frac{\lambda}{\alpha+2\delta} M^* \tag{41}$$

$$\frac{dT}{dt}=0 = \alpha L^* = \mu T^* \tag{42}$$

$$T^* = \frac{\alpha}{\mu} = L^* = \frac{\alpha}{\mu(\alpha+2\delta)} \lambda M^* \tag{43}$$

$$\frac{dD}{dt}=0 = \alpha L^* = \mu T^* \tag{44}$$

$$D^* = \frac{\delta}{\mu} M^* + \frac{2\delta}{\mu} L^* \tag{45}$$

$$D^* = \frac{\delta}{\mu} M^* + \frac{2\delta}{\mu} \frac{\lambda}{\alpha+2\delta} M^* \tag{46}$$

Equation for U^* at the endemic equilibrium point:

$$\frac{dU}{dt}=0 = T - (\beta M^* + \tau L^*)U^* - \mu U^* \tag{47}$$

$$U^* = \frac{T}{\mu+\beta M^*+\tau L^*} \tag{48}$$

Substituting L^* into the equation:

$$U^* = \frac{T}{\mu+\beta M^*+\tau\frac{\lambda}{\alpha+2\delta}M^*} \tag{49}$$

$$U^* = \frac{T}{\mu+M^*\left[\beta+\frac{\tau\lambda}{\alpha+2\delta}\right]} \tag{50}$$

For M^*:

$$0 = \beta U^* M^* + \tau U^* L^* - (\lambda+\delta) M^* \tag{51}$$

$$0 = U^* M^* \left[\beta + \tau \frac{L^*}{M^*}\right] - (\lambda+\delta) M^* \tag{52}$$

$$0 = M^* \left[U^*\left(\beta + \frac{\tau\lambda}{a+2\delta}\right) - (\lambda+\delta)\right] \tag{53}$$

since the endemic equilibrium satisfies $M^*>0$:

$$U^*\left(\beta+\frac{\tau\lambda}{a+2\delta}\right) = (\lambda+\delta) \tag{54}$$

$$U^* = \frac{\lambda+\delta}{\beta+\frac{\tau\lambda}{a+2\delta}} \tag{55}$$

Let us equate this value of U with the previously obtained U^*:

$$R_E = \left(\frac{\beta}{\lambda+\delta} + \frac{\tau}{(\lambda+\delta)(a+2\delta)}\right) \tag{56}$$

This shows that the endemic equilibrium exists only if $R_E>0$

$$U^* = \frac{k_2}{k_1} \tag{57}$$

$$M^* = \frac{T-\mu U^*}{U^* k_1}$$

$$L^* = \frac{\lambda}{k_3} M^*$$

$$T^* = \frac{\alpha}{\mu} L^*$$

$$D^* = \frac{\delta}{\mu} M^* + \frac{2\delta}{\mu} L^*$$

$$k_1 = \beta + \frac{\tau\lambda}{a+2\delta},\ k_2 = \lambda+\delta,\ k_3 = a+2\delta,\ k_4 = \mu \tag{58}$$

LEMMA 1: If $R_E<1$, then the equilibrium point E_0 which is free of students' learning behavior, is asymptotically stable.

First, let us write the Jacobian matrix of the system.

$$J_{E_0} = \begin{vmatrix} -\beta M - \tau L - \mu & -\beta U & -\tau U & 0 & 0 \\ \beta M + \tau L & \beta U + \tau L - (\lambda+\delta) & \tau U & 0 & 0 \\ 0 & \lambda & -(a+2\delta) & 0 & 0 \\ 0 & 0 & \alpha & -\mu & 0 \\ 0 & \delta & 2\delta & 0 & -\mu \end{vmatrix} \tag{59}$$

At the free equilibrium,. The characteristic equation of Figure (1).

$$J_{E_0} = \begin{vmatrix} -\mu & -\beta\frac{T}{\mu} & -\tau\frac{T}{\mu} & 0 & 0 \\ 0 & \beta\frac{T}{\mu} + \tau L - (\lambda+\delta) & \tau\frac{T}{\mu} & 0 & 0 \\ 0 & \lambda & -(a+2\delta) & 0 & 0 \\ 0 & 0 & \alpha & -\mu & 0 \\ 0 & \delta & 2\delta & 0 & -\mu \end{vmatrix} \tag{60}$$

As a result of the cofactor expansion of the matrix.

$$FV^{-1} = \begin{vmatrix} \frac{\beta T/\mu}{\lambda+\delta} & \frac{\tau T/\mu}{a+2\delta} \\ \frac{\lambda}{\lambda+\delta} & 0 \end{vmatrix}$$

$$\lambda^2 - \lambda\left(\frac{\beta T}{\mu(\lambda+\delta)} + \frac{\tau T}{\mu(\lambda+\delta)(a+2\delta)}\right) = 0 \quad (61)$$

The resulting eigenvalues are: $\lambda_1 = 0$ ve $\lambda_2 = \left(\frac{\beta T}{\mu(\lambda+\delta)} + \frac{\tau T}{\mu(\lambda+\delta)(a+2\delta)}\right) = R_E$. Therefore, if 1 then and the disease-free equilibrium is unstable.

İf $R_E < 1$, the eigenvalues are negative and the disease-free equilibrium is stable (Figure 4).

If we evaluate the stability of the Jacobian at the endemic equilibrium point.

$$J_{E^*} = \begin{vmatrix} -\beta M^* - \tau L^* - \mu & -\beta U^* & -\tau U^* & 0 & 0 \\ \beta M^* + \tau L^* & k_1 U^* - k_2 & \tau U^* & 0 & 0 \\ 0 & \lambda & -k_3 & 0 & 0 \\ 0 & 0 & \alpha & -\mu & 0 \\ 0 & \delta & 2\delta & 0 & -\mu \end{vmatrix} \quad (62)$$

For a system to be asymptotically stable, all eigenvalues must satisfy Re(λ) < 0. However, since solving the eigenvalues individually is difficult, the Routh–Hurwitz criterion determines stability directly from the coefficients.

$$S_1 = c_1 > 0, S_2 = c_2 c_1 - c_0 > 0, S_3 = c_0 > 0 \quad (63)$$

If these three conditions are satisfied→ then the endemic equilibrium is asymptotically stable.

The characteristic equation obtained at the endemic equilibrium point.

$$\lambda^3 + c_2\lambda^2 + c_1\lambda + c_0 = 0$$

$$c_2 = k_3 + k_2 - k_1 U^*$$

$$c_1 = k_3(k_2 - k_1 U^*) + \lambda\tau U^*$$

$$c_0 = k_3 k_2 - k_3 k_1 U^*)$$

$$k_1 = \beta + \frac{\tau\lambda}{a+2\delta}, k_2 = \lambda + \delta, k_3 = a + 2\delta \quad (64)$$

All equilibrium values are positive. This confirms that the existence of the endemic equilibrium is determined by the condition $R_E > 1$. Therefore, the endemic equilibrium is stable.

III. Results and Dıscussıon

The cellular automaton simulation presented in Figure 2 illustrates in detail how learning processes evolve over time across four fundamental student states. At the beginning of the simulation, the number of uninformed individuals decreases rapidly, while the learning and low-motivation groups exhibit a marked increasing trend. In particular, low-motivation students begin to form the largest group in the medium term as the interaction network becomes denser, indicating that motivation loss emerges as a dominant dynamic within the system.Although the learning group initially shows an increase, it later follows a fluctuating yet nearly horizontal trajectory, suggesting that learning behavior persists in the population but at a limited level. The motivated group, on the other hand, maintains a relatively low density and displays a stable distribution over time, implying that the spread rate of motivation is weaker compared to other transition processes.Overall, the graph demonstrates that learning and motivation dynamics are shaped within a nonlinear interaction network. In particular, it reveals that motivation loss plays a critical role in determining the long-term behavior of the system.

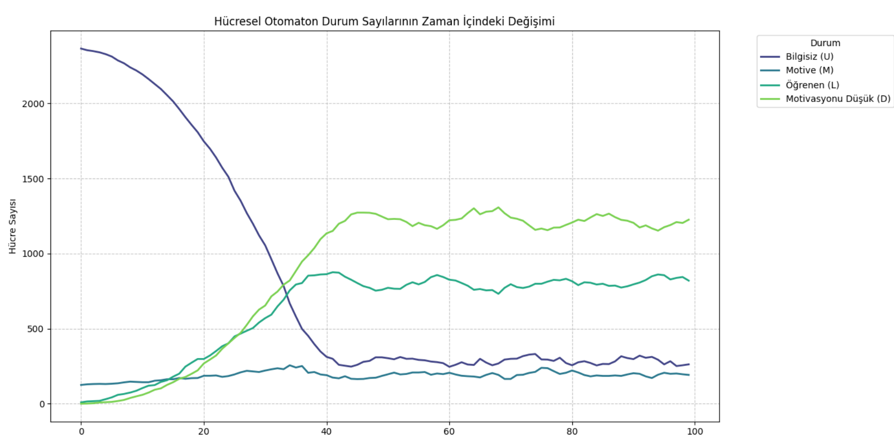

Figure 2 Cellular Automaton Simulation Graph of Learning Processes over Time (Seconds)

Figure 3 illustrates the change in motivation level over time (seconds) within the extended SIR model, enabling a clearer understanding of the system's dynamic structure. In the initial phase of the graph, a rapid decline in motivation level is observed, indicating the strong influence of the motivation loss parameter and showing that a significant portion of students struggle to maintain motivation during the early stages of the learning process.

In the medium term, the motivation level remains low and relatively stable for an extended period, suggesting that the system operates within a weak interaction and learning diffusion regime during this phase. However, after approximately the 130th step, a noticeable acceleration in motivation emerges, and from around the 150th step onward, the process reaches an almost maximum level and stabilizes at a saturation point. This late-stage increase indicates that interaction parameters that enhance motivation (such as teacher influence or peer support) become dominant, revitalizing the learning process.

Overall, the graph clearly demonstrates that motivation is influenced not only by initial conditions but also by the combined effects of evolving social and cognitive interactions over time, resulting in a distinctly nonlinear growth pattern.

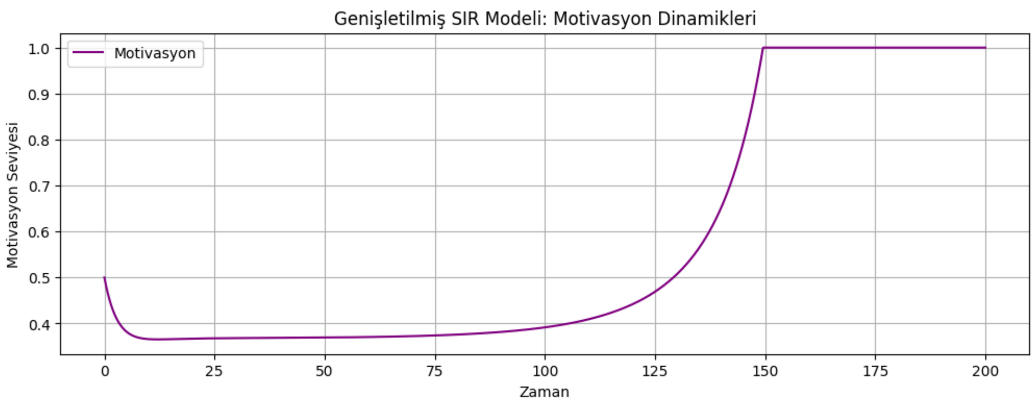

Figure 3 Motivation dynamics

IV. Conclusıon

In this study, an extended SIR model integrated with a cellular automaton structure was used to examine students' processes of knowledge acquisition, motivation development, and mastery within a mathematical and computational framework, aiming to understand the dynamic nature of learning processes. The model represents a complex learning ecosystem in which students are categorized into five distinct states uninformed, motivated, learning, mastered, and low motivation and transitions between these states are governed by parameters such as social interaction, motivation, teacher influence, and peer support.

Analytical solutions derived from different systems of differential equations revealed the behavior of the model at both the purified equilibrium point and the endemic equilibrium point. In particular, it was shown that a threshold value analogous to the basic reproduction number determines whether learning becomes persistent within the population. This result indicates that learning behavior, similar to epidemiological spread, can be sustained only when certain critical thresholds are exceeded.

Cellular automaton simulations supported the theoretical findings by clearly demonstrating the nonlinear nature of learning and motivation processes. Simulation results showed that motivation loss emerges as a dominant dynamic over time, while the motivated and learning groups remain below a critical threshold. This finding highlights the crucial role of the learning environment and interaction intensity in sustaining motivation, emphasizing that teacher influence, peer support, and appropriately designed learning tasks are key factors in reactivating motivation. Indeed, the sharp rise in motivation dynamics after a prolonged low-level phase indicates that learning ecosystems are highly sensitive to external interventions or structural changes.

Student learning behavior dynamics were examined within the framework of an extended SIR-like model. The model consists of five compartments: uninformed (*U*), motivated (*M*), learning (*L*), practicing/mastered (*T*), and low-motivation (*D*) individuals. System behavior was evaluated by considering the basic reproduction number and the stability of endemic and purified equilibrium points.

Mathematical analysis revealed that when $R_E < 1$ learning behavior does not spread within the population and the system converges toward the purified equilibrium. When $R_E > 1$ learning behavior becomes persistent and the system moves toward the endemic equilibrium. Although the proportion of uninformed individuals initially decreases slightly, it stabilizes over time at a relatively high and steady level. The levels of motivated and learning individuals rapidly decline and stabilize near zero. The practicing/mastered and low-motivation groups remain at low levels, indicating that learning behavior cannot be sustained in the population under these conditions Motivation levels show a significant decline during the initial phase and gradually settle at a very low but stable value over time. This outcome indicates that the parameters λ, τ, and are insufficient to support learning diffusion in a low-interaction environment. The Routh–Hurwitz conditions $S_1 > 0$, $S_2 > 0$, $S_3 = 0$ confirm that the purified equilibrium point is stable when $R_E < 1$. Thus, both analytically and numerically, the system converges to a learning-free equilibrium under low transmission coefficients. Learning behavior spreads only when parameters exceed a critical threshold; otherwise, it weakens and disappears.

This study demonstrates that student learning behavior dynamics can be mathematically modeled and highlights the critical influence of specific interaction parameters on learning diffusion. Overall, by applying an extended SIR model to learning processes, this research contributes both theoretically and practically to educational research. The model quantitatively represents the diffusion of learning motivation, peer effects, and teacher interventions, enabling a deeper understanding of complex social processes in educational environments Moreover, through simulation and analytical approaches, the study provides insights into the conditions required for sustainable learning and offers practical implications for instructional design, learning analytics, and motivation support systems. The proposed framework lays the groundwork for future research directions, including testing alternative parameter combinations, integrating individual differences into the model, and analyzing similar processes in artificial intelligence–based learning environments. As such, the model can be considered a powerful tool to guide both theoretical expansion and applied educational science research.

Conflict of Interest

The authors declare that there is no conflict of interest.

Author Contributions

The authors declare that they contributed equally to this work.

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